## International prize “Tullio Levi-Civita” for the Mathematical and Mechanical Sciences

We are pleased to announce that the *2024 Levi-Civita Prize for the Mathematical and Mechanical Sciences* will be presented to

# BORIS KOLEV

for the quality, the depth, the innovative character and international value of over 50 years of activity in Theoretical Mechanics.

The mathematical trajectory of Boris Kolev represents exactly what the Levi-Civita Prize honors: the elegant understanding of mechanics by using deep differential geometry. Boris does not merely apply mathematical machinery to physical problems; he manages to uncover the intrinsic geometric architecture which was already there, hidden beneath coordinate layers and dense tensorial equations.

Three are the major pillars of his scientific career.

**1. Infinite-Dimensional Differential Geometry and Hydrodynamics**

The first is his seminal work on the geometric foundations of fluid dynamics, carrying forward the modern tradition of Vladimir Arnold, David Ebin, and Jerrold Marsden. Boris brought incredible clarity to the study of the Euler equations and related non-linear systems by viewing them as geodesic flows on infinite-dimensional Lie groups.

In his highly regarded work, « On the geometric approach to the motion of inertial mechanical systems » (Constantin & Kolev, 2002) and « Geodesic flow on the diffeomorphism group of the circle » (Constantin & Kolev, 2003), he provided a masterclass on how Euler equations on a Lie group describe the motion of physical systems—ranging from the rigid body to the complex hydrodynamics of perfect fluids. His papers on the rigorous definition of the exponential map and when this map can serves as a local chart as it is the case in finite dimensionnal Riemmannian geometry solved deep analytical hurdles.

In « The Degasperis-Procesi equation as a non-metric Euler equation » (Escher and Kolev, 2011), he extended this formalism to the case of *non-metric* Euler-Arnold equations, such as the *Degasperis-Procesi equation* which cannot be described as the geodesic equation of a Riemannian metric, but only by an affine covariant derivative.

Crucially, in « The geometry of a vorticity model equation » (Escher et al., 2012), he realizes that integer Sobolev metrics where not sufficient to modelize, for instance, the geostrophic approximation of fluid dynamics and extended in "Right-invariant Sobolev metrics of fractional order on the diffeomorphism group of the circle" (Escher & Kolev, 2014) and « Local and global well-posedness of the fractional order EPDiff equation on R*d* » (Bauer et al., 2015), this geometric formulation to non-local inertia operators. This allowed him to prove the well-posedness and smoothness of the Riemannian exponential map for fractional Sobolev norms s >= ½, and later to some pseudo-differential inertia operators on compact manifolds (Bauer et al., 2020).

Furthermore, his studies of shallow water waves, such as in "Two-component equations modelling water waves with constant vorticity" (Escher et al., 2014), demonstrated how the physical phenomenon of breaking waves or "peakons" could be mathematically mapped as pristine geodesic paths on curved manifolds. Where others saw non-linear partial differential equations prone to blow-up, Boris provided structural order through infinite-dimensional Riemannian metrics.

## 2. Anisotropy and the Algebraic Structure of Continuum Mechanics

The second pillar is his work on material symmetries and anisotropy in continuum mechanics. For decades, the mechanics and engineering communities grappled with the structural description of higher-order tensors—specifically the fourth-order elasticity tensor (C) and related piezoelectric or photoelastic tensors. The classic problem was defining a sharp, coordinate-independent classification of symmetry classes.

Boris approached this problem by leveraging classical invariant theory and the algebraic geometry of group representations. In the breakthrough paper, "A minimal integrity basis for the elasticity tensor" (Auffray, Olive & Kolev, 2017), he solved with his student Marc Olive a historical challenge by obtaining, for the very first time, a complete, minimal integrity basis consisting of 297 polynomial invariants for the elasticity tensor. This was achieved by mapping harmonic tensors in R^3 to binary forms and implementing Gordan's classical algorithm.

He expanded on this structural architecture in "Harmonic Factorization and Reconstruction of the Elasticity Tensor" (Olive, Kolev et al., 2018), showing how a complex, coupled mechanical response can be systematically split into irreducible, invariant pieces. Rather than stopping at pure theory, he applied these invariants to longstanding engineering puzzles. In "The Distance to Cubic Symmetry Class as a Polynomial Optimization Problem" (Kolev & Olive, 2020), he provided a quasi-analytical solution using Gröbner bases to determine exactly how far a raw, experimentally measured material tensor deviates from an ideal symmetry stratum.

### 3. Geometric Foundations of Continuous Mechanics

The third pillar highlights his capacity to unify classical and modern paradigms. In « Objectives rates as covariant derivatives on the manifold of Riemannian metrics » (Kolev & Desmorat, 2024), he provides for the first time the true nature of objective derivatives as covariant derivatives on the infinite-dimensional manifold of Riemannian metrics on the body.

This leads him to investigate more deeply the formulation of Continuum Mechanics in the framework of General Relativity. Not for its own sake, but to better understand the definition of strain and stress and the Principle of Virtual Work in this context, with the goal to better formulate the coupling of solid mechanics with electromagnetism (Kolev and Desmorat, 2023).

In his recent paper, "Classical and relativistic balance of configurational forces" (Kolev et al., 2025), Boris revisited the mechanics of material forces. By treating the reference configuration of a material body as an abstract three-dimensional manifold and executing variations directly on the reference embedding, he provided an unassailable geometric proof that the balance of configurational forces—the Eshelby stress balance—is not an independent physical postulate, but an elegant consequence of standard momentum balance combined with constitutive covariance. He seamlessly extended this framework to Special Relativity, showing how the 4D Eshelby tensor emerges naturally from the Noether stress-energy tensor.

Finally he has been active to try to reconnect the mechanical community with the mathematical community through the creation (with Aziz Hamdouni) of the GDR « Differential Geometry and Mechanichs » and through the participation to several lectures in Masters and Doctoral Schools in Mechanics.

**The Style of a Scholar**

Beyond the theorems:

Boris is must be appreciated for his intellectual generosity. His papers are masterclasses in exposition. He has an uncommon gift for taking heavy, intimidating algebraic machinery—whether it is the theory of SO(3) representations, the technicalities of Fréchet manifolds, or the Haar measure—and rendering it accessible, functional, and deeply elegant for the mechanics community.

He does not write to obscure; he writes to illuminate. In doing so, he has reminded us all that mechanics is not merely a collection of empirical balances, but a beautiful branch of geometry.

For his profound contributions to the geometric formulation of hydrodynamics, his elegant resolution of the algebraic structure of anisotropy, and his role as a vital bridge-builder between pure mathematics and continuum physics, we are proud to present the *Levi-Civita Prize* to Boris Kolev.

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